COLOR, SPIN, AND FLAVOR-DEPENDENT FORCES IN QUANTUM CHROMODYNAMICS

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In Memoriam Kenneth A. Johnson 1931–1999

A simple generalization of the Breit Interaction explains many qualitative features of the spectrum of hadrons.　　　hep-ph/0001123　　　MIT CTP # 2938

1　Introduction

I did not know Gregory Breit, but two of his closest associates influenced me deeply. I did research in Gerry Brown's group as an undergraduate at Princeton. Gerry welcomed young students into his research family and gave us wonderful problems to work on. He taught us that complicated problems often have simple answers – something that has proved true in particle physics over the past quarter century. Vernon Hughes invented deep inelastic spin physics in the early 1970's and has championed it relentlessly ever since. He and his collaborators have literally rewritten the book on the quark and gluon structure of the nucleon. It is a pleasure to speak at a symposium celebrating their teacher, Gregory Breit.

My talk will be largely pedagogical. I would like to show how a generalization of the "Breit Interaction" can account for some of the regularities of hadron physics. Although the basic ideas described here date back to the 1970's, they have not been presented quite this way before, and some developments are still at the forefront of modern research in QCD. This subject was a special favorite of my friend and collaborator, Ken Johnson, who died this past winter. Ken had many friends at Yale. He would have liked to hear this story, so I dedicate my talk to his memory.

It is hard to make definite statements about hadrons made of the light u, d, and s quarks. The nonperturbative regime is too complicated. It may never be solved to our satisfaction except on a computer. Nevertheless, the spectrum and interactions of baryons and mesons display remarkable regularities, which correlate with simple symmetry properties of the fundamental quark/gluon interactions. The role of models in QCD is to build simple physical pictures that connect the phenomenological regularities with the underlying structure.

The QCD "Breit Interaction" is the spin-dependent part of one-gluon exchange between light quarks in the lowest state of some unspecified mean field. It is summarized by an effective Hamiltonian acting on the quarks' spin and color indices,

$$\mathcal{H}_{\text{eff}} \propto -\sum_{i \neq j} \underset{\sim}{\lambda}_i \cdot \underset{\sim}{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \qquad (1)$$

where $\vec{\sigma}_i$ and $\underset{\sim}{\lambda}_i$ are the spin and color operators of the ith quark. The spin operators are represented by the three 2×2 Pauli matrices, normalized to $\text{Tr}(\sigma_i^k)^2 = 2$ for $k = 1, 2, 3$, and the color operators are represented by the eight 3×3 Gell-Mann matrices, normalized the same way, $\text{Tr}(\lambda_i^a)^2 = 2$ for $a = 1, \ldots 8$. The sum over i and j extends over all quark pairs. For the moment, I ignore antiquarks. I also ignore quark mass differences. In reality the u and d masses are small enough relative to the natural scale of QCD that we can neglect them. The s quark is heavier. It is a reasonable first approximation to ignore its mass as well. The c, b, and t quarks are too heavy, and cannot be treated this way. The space-time dependence of $\mathcal{H}_{\text{eff}}$ is not well understood, but in this approximation it is universal, and need not concern us much.

$\mathcal{H}_{\text{eff}}$ can be read off the Feynman diagram for one-gluon exchange between quarks; see Fig. (1). At short distances where QCD is weakly coupled, we can trust perturbation theory, and one-gluon exchange should dominate. However, at typical hadronic distance scales QCD is strongly coupled. Still, there is reason to take the qualitative predictions which follow from $\mathcal{H}_{\text{eff}}$ seriously. Once the long range, spin-average, confining interactions in QCD have been integrated out, the resulting confining, bag-like mean field acts as an infrared cutoff, reducing the strength of the remaining QCD effects. Most phenomenological models of QCD – the Bag Model, the nonrelativistic quark model, and other quark models in particular – use this picture successfully. The absence of strong renormalization (higher twist effects) in deep inelastic scattering offers phenomenological support for a picture of hadrons where perturbation theory is qualitatively reliable once confinement has been implemented.

A couple of further notes on the form of eq. (1): First, the spin-averaged piece of one-gluon exchange has been set aside. It figures in the dynamics of confinement but not in the spectroscopy considered here. Second, the tensor and spin-orbit interactions generated by one-gluon exchange average to zero in the lowest quark state. Third, the appearance of $\vec{\sigma}$ matrices in eq. (1) does not mean the analysis is nonrelativistic. For light quarks this would be an unacceptable restriction. The Dirac $\vec{\alpha}$ matrices which appear in the relativistic quark currents reduce to $\vec{\sigma}$ matrices in the lowest orbital.
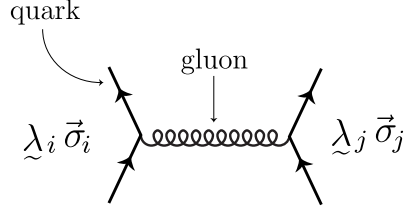
Figure 1. One-gluon exchange between quarks.

For obvious reasons the interaction of eq. (1) is known as the "colorspin" or "color-magnetic" interaction of QCD.

$\mathcal{H}_{\text{eff}}$ was first introduced by De Rujula, Georgi, and Glashow in their pioneering paper on hadron spectroscopy in QCD.[1] Many of the spectroscopic results I will discuss were developed by them or by our group at MIT in the mid-1970's.[2,3,4] The subject of the scalar mesons has been revitalized recently by Schechter and his collaborators.[5] The possible role of $\mathcal{H}_{\text{eff}}$ in quark matter has been the subject of much recent activity starting with the fundamental work of Alford, Rajagopal, and Wilczek.[6]

My talk is organized as follows: In Section 2 I review the basic symmetry structure of $\mathcal{H}_{\text{eff}}$. In Section 3 I look at some properties of the baryons: the octet-decuplet splitting, the Λ–Σ splitting, and the pattern of excitations. Section 4 is devoted to mesons: first the pseudoscalar-vector splittings, next a remark on the absence of exotics, and finally a reevaluation of the $J^{PC} = 0^{++}$ mesons. In Section 5 I return to symmetry and extract a simple rule for the ground state of the Q^N configuration – the rule of flavor antisymmetry. In Section 6, I apply it to baryons (Q^3) and dibaryons (Q^6). Finally, in Section 7 I give a very brief introduction to the effects of $\mathcal{H}_{\text{eff}}$ in quark matter – condensates, superconductivity, and unusual patterns of symmetry breaking.

2   Basics: Regularities of the Q–Q interaction

It is not necessary to use much mathematics to understand the implications of eq. (1) for the simplest case of two quarks. Most of what I need can be done merely by rewriting it in terms of color, spin, and flavor "exchange operators". The spin exchange operator, P_{12}^S, is defined by

$$P_{12}^S = \tfrac{1}{2} + \tfrac{1}{2}\vec{\sigma}_1 \cdot \vec{\sigma}_2 \ . \tag{2}$$

Two spin-$\frac{1}{2}$ particles may be coupled to a triplet of spin-1 states, $|3_\mathrm{s}\rangle \equiv \{|\uparrow\uparrow\rangle, \frac{1}{\sqrt{2}}|\uparrow\downarrow + \downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$, which are symmetric under spin exchange, or to a singlet spin-0 state, $|1_\mathrm{s}\rangle \equiv \frac{1}{\sqrt{2}}|\uparrow\downarrow - \downarrow\uparrow\rangle$, which is antisymmetric under spin exchange.[*] The eigenvalues of $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ are $+1$ and -3 in the spin-1 and spin-0 states respectively, so $P^S|3_\mathrm{s}\rangle = +|3_\mathrm{s}\rangle$, $P^S|1_\mathrm{s}\rangle = -|1_\mathrm{s}\rangle$. Thus P^S has the desired property that it gives ± 1 on states which are symmetric/antisymmetric under spin exchange.

Color and flavor states are both classified by $SU(3)$, so color and flavor exchange operators have the same form. To be specific, consider flavor. The quark labels are u, d, and s. I need a notation for the set of Gell-Mann matrices associated with flavor – denote them by $\{\underset{\sim}{\beta}\}$ to distinguish them from the $\{\underset{\sim}{\lambda}\}$ used for color.

There are nine flavor states of two quarks. Six are symmetric: $|6_\mathrm{f}\rangle \equiv \{|uu\rangle, |dd\rangle, |ss\rangle, \frac{1}{\sqrt{2}}|ud + du\rangle, \frac{1}{\sqrt{2}}|ds + sd\rangle, \frac{1}{\sqrt{2}}|su + us\rangle\}$. Three are antisymmetric: $|\bar{3}_\mathrm{f}\rangle \equiv \{\frac{1}{\sqrt{2}}|ud - du\rangle, \frac{1}{\sqrt{2}}|ds - sd\rangle, \frac{1}{\sqrt{2}}|su - us\rangle\}$. These states can be visualized most easily in the usual $SU(3)_\mathrm{f}$ weight diagram in which the third component of isospin ($I_3 = \frac{1}{2}(n_u - n_d)$) is the x-axis and hypercharge ($Y = \frac{1}{3}(n_u + n_d - 2n_\mathrm{s})$) is the y-axis. The original quark triplet forms a triangle, point down, the 3_f. The antisymmetric set of two quark states forms a triangle, point up, hence the notation $\bar{3}_\mathrm{f}$. The symmetric set of two quark states form a larger triangle, point down, the 6_f. All three representations are shown in Fig. (2).

A simple exercise with the Gell-Mann matrices lead to the exchange operator for flavor,

$$P^F_{12} = \tfrac{1}{3} + \tfrac{1}{2}\underset{\sim}{\beta}_1 \cdot \underset{\sim}{\beta}_2 \tag{3}$$

with the desired property that $P^F|6_\mathrm{f}\rangle = +|6_\mathrm{f}\rangle$ and $P^F|\bar{3}_\mathrm{f}\rangle = -|\bar{3}_\mathrm{f}\rangle$. Since color has the same structure as flavor:

$$P^C_{12} = \tfrac{1}{3} + \tfrac{1}{2}\underset{\sim}{\lambda}_1 \cdot \underset{\sim}{\lambda}_2 \tag{4}$$

and $P^C|6_\mathrm{c}\rangle = +|6_\mathrm{c}\rangle$, $P^C|\bar{3}_\mathrm{c}\rangle = -|\bar{3}_\mathrm{c}\rangle$.

Armed with eqs. (2), (3), and (4), we can rewrite the interaction between two quarks in terms of exchange operators,

$$\mathcal{H}_\mathrm{eff} \propto -\underset{\sim}{\lambda}_1 \cdot \underset{\sim}{\lambda}_2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 = -4P^C_{12}P^S_{12} + \tfrac{4}{3}P^S_{12} + 2P^C_{12} - \tfrac{2}{3}. \tag{5}$$

The next step exposes the reason for all this algebra: two quarks in the same orbital are *symmetric* under space exchange. Since the quarks are

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[*]To avoid confusion, we label all states by their degeneracy. Thus the spin-1 states are $|3_\mathrm{s}\rangle$ and the spin-0 states are $|1_\mathrm{s}\rangle$.
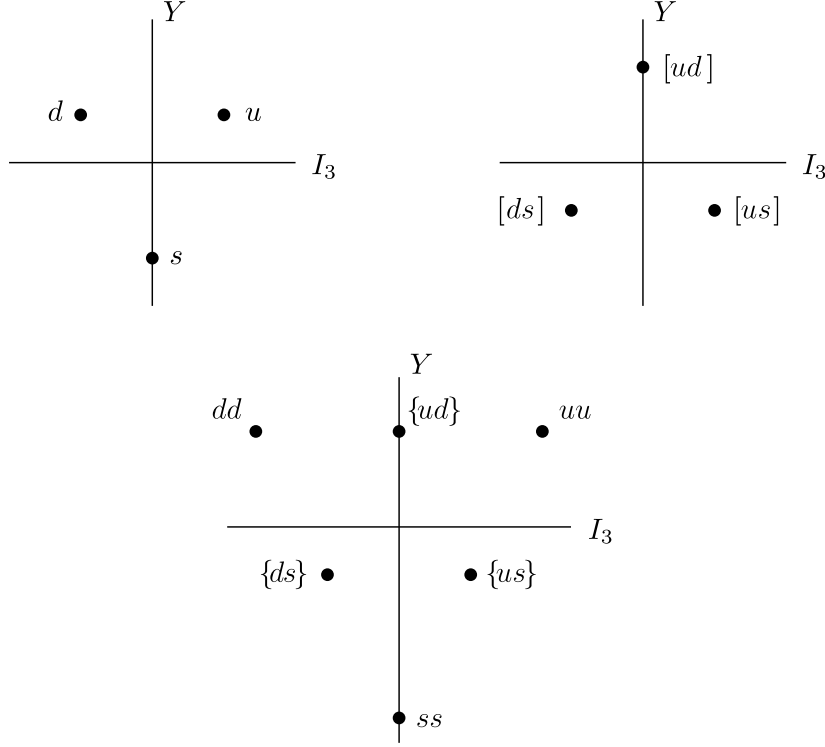
Figure 2. $SU(3)_f$ weight diagram for the fundamental 3_f, antisymmetric diquark, $\bar{3}_f$, and symmetric diquark, 6_f representations.

fermions, the wavefunction must be *antisymmetric* under the simultaneous exchange of the remaining labels: spin, color, and flavor. In terms of exchange operators,

$$P_{12}^C P_{12}^S P_{12}^F = -1 \qquad \text{or}$$
$$P_{12}^C P_{12}^S = -P_{12}^F \,. \tag{6}$$

So eq. (5) can be rewritten as

$$\mathcal{H}_{\text{eff}} = 4P_{12}^F + \tfrac{4}{3}P_{12}^S + 2P_{12}^C - \tfrac{2}{3} \,. \tag{7}$$

Lo and behold: The dominant piece of the colorspin force (weighted by 4) is a *flavor*-exchange interaction. *This is why colorspin has significant implications for flavor-dependent mass splittings.*

Table 1. Flavor, spin, and color states of two quarks, and the eigenvalue of the QCD Breit Interaction of eq. (1).

Flavor	Spin	Color	ΔE
$\bar{3}(A)$	$1(A)$	$\bar{3}(A)$	-8
$\bar{3}(A)$	$3(S)$	$6(S)$	$-4/3$
$6(S)$	$3(S)$	$\bar{3}(A)$	$8/3$
$6(S)$	$1(A)$	$6(S)$	4

There are four totally antisymmetric configurations of flavor×spin×color. They and their associated colorspin interaction strengths are listed in Table 1. From eq. (7) it is clear that the configuration separately antisymmetric in flavor, spin, and color, is the most attractive channel. The other three configurations are symmetric in at least two indices, leading to dramatically less binding. The dominance of a single qq configuration, $|\bar{3}_f 1_s \bar{3}_c\rangle$, is the basic observation behind colorspin phenomenology. Colorspin favors antisymmetric (and therefore low-dimension) representations of the flavor and spin symmetry groups. Several applications will bear this out.

3 Baryons

3.1 The Octet-Decuplet Mass Difference

The lightest baryons composed predominantly of u, d, and s quarks can be grouped into an $SU(3)_f$ octet with spin-$\frac{1}{2}$ and an $SU(3)_f$ decuplet with spin-$\frac{3}{2}$. The $SU(3)_f$ weight diagrams in terms of I_3 and Y are shown in Fig. (3). States with the same I_3 and Y contain the same valence quarks. Thus the Δ^+ and p must both contain uud in addition to any $\bar{q}q$ pairs and gluons. For all pairs like the $p - \Delta^+$, the state in the decuplet is roughly 200–250 MeV heavier than the equivalent state in the octet. The *sign* of this effect is easily explained from the symmetry properties of the colorspin force: Since baryons are color singlets, each quark pair must be coupled to a color $\bar{3}_c$.[†] Because the decuplet has spin-$\frac{3}{2}$, each quark pair must be coupled to a spin triplet. Referring to Table 1, it appears that the $\bar{3}_c 3_s$ colorspin configuration has positive interaction energy. The spin coupling of the quarks in the octet is more complicated, but one thing is clear: at least the coupling of quark pairs

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[†]So a color singlet can be made when the third quark is included: $\bar{3} \otimes 3 = 1 \oplus 8$, whereas $6 \otimes 3 = 8 \oplus 10$.
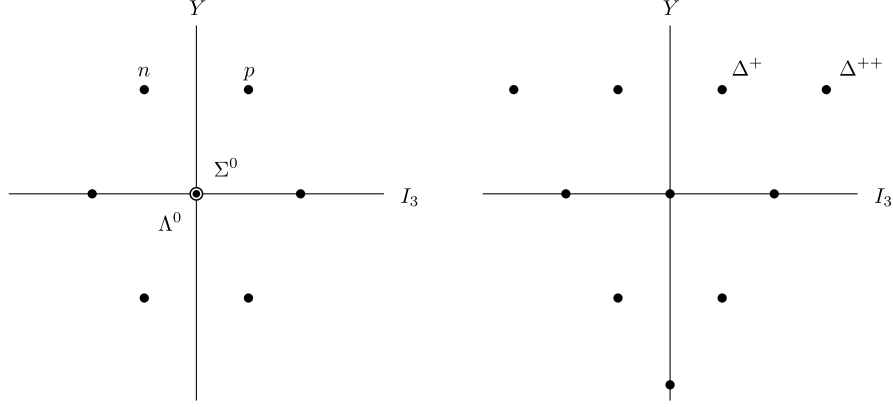
Figure 3. $SU(3)_{\mathrm{f}}$ weight diagram for the baryon octet and decuplet. Some states discussed in the text are labeled.

to spin-0 is allowed. Therefore a state in the octet must be lighter than state with the same quark content in the decuplet.

3.2 The Λ–Σ Splitting

The lightest neutral strange baryons, the $\Lambda^0(1116)$ and the $\Sigma^0(1192)$ have the same quark content, uds. They differ in mass by 76 MeV. Before QCD this mass difference puzzled theorists who tried to understand the quark structure of hadrons. The puzzle was resolved by DeRujula, Georgi, and Glashow and led to much work on QCD inspired quark models.[1] The Λ^0 and the Σ^0 have the same quarks and the same spin. They differ in isospin:

$$|\Sigma^0\rangle \sim |(ud)^{I=1}s\rangle \qquad |\Lambda^0\rangle \sim |(ud)^{I=0}s\rangle \ . \qquad (8)$$

Colorspin, like all spin-dependent effects in gauge theories, is a relativistic effect. For heavy quarks the coefficient we have suppressed in front of $\mathcal{H}_{\mathrm{eff}}$ goes like $1/m_q^2$. It is reasonable to assume that colorspin forces weaken with increasing quark mass. The u and d quarks are much lighter than the s quark, so the colorspin force between a ud pair is stronger than that between a us or ds pair. The ud pair in the Λ^0 is in the $I = 0$ or $\bar{3}_{\mathrm{f}}$ state. In the Σ^0, the ud pair is in the $I = 1$ or 6_{f} state. The ud colorspin contribution to the mass is therefore attractive in the Λ^0, repulsive in the Σ^0. Since this dominates over ds and us, the Λ^0 must be lighter than the Σ^0. The magnitude of the $\Sigma^0 - \Lambda^0$ mass difference depends on the strength of the colorspin interaction (fixed by the decuplet-octet splitting) and the quark mass differences (fixed by $SU(3)_{\mathrm{f}}$

	Flavor × Spin	Space

$56_{\mathrm{fs}} \supset 8_{\mathrm{f}}2_{\mathrm{s}} \oplus 10_{\mathrm{f}}4_{\mathrm{s}}$

$70_{\mathrm{fs}} \supset 1_{\mathrm{f}}2_{\mathrm{s}} \oplus 8_{\mathrm{f}}2_{\mathrm{s}} \oplus 8_{\mathrm{f}}4_{\mathrm{f}} \oplus 10_{\mathrm{f}}2_{\mathrm{s}}$

$20_{\mathrm{fs}} \supset 1_{\mathrm{f}}4_{\mathrm{s}} \oplus 8_{\mathrm{f}}2_{\mathrm{s}}$

Figure 4. States of three quarks. $SU(6)_{\mathrm{fs}}$ multiplets are shown along with their $SU(3)_{\mathrm{f}} \otimes SU(2)_{\mathrm{s}}$ content. The multiplets are represented by Young Diagrams. Boxes stacked vertically represent antisymmetrized quarks, those stacked horizontally are symmetrized. To make states symmetric under flavor×spin×space, quarks must have the same permutation symmetry under flavor×spin and space. Hence the identical Young Diagrams.

violating mass differences). The numerical value is roughly consistent with the 76 MeV observed.

3.3 Excited Baryon Spectroscopy

Tables of the masses and properties of excited baryons fill books.[7] Long ago, Dalitz and collaborators[8] showed that the fundamental structure of this spectrum could be understood in a "Symmetric Quark Model": Populate spin×flavor×space symmetric states of three quarks (color antisymmetry takes care of Fermi statistics) in the modes of some simple central potential. The lightest (positive parity) baryons have all quarks in the lowest orbital. To obtain negative parity excitations, promote one quark to the first ($L = 1$) excited orbital. To make positive parity excitations, promote two quarks to the first level or one quark to the second.

Dalitz's model contains the Gürsey's famous $SU(6)_{\mathrm{fs}}$ symmetry[‡] built out of $SU(3)_{\mathrm{f}} \otimes SU(2)_{\mathrm{s}}$.[9] Baryon multiplets form representations of $SU(6)_{\mathrm{fs}}$, which in turn contain $SU(3)_{\mathrm{f}}$ multiplets of definite spin. The three $SU(6)_{\mathrm{fs}}$ representations of three quarks are shown in Fig. (4) in terms of Young Diagrams. The 56_{fs} is totally symmetric in exchange of spin and flavor, the 70_{fs} has mixed symmetry, and the 20_{fs} is totally antisymmetric. The 56_{fs} and 70_{fs} are prominent in the baryon spectrum. For example, the lightest baryons form a 56_{fs} containing the decuplet with spin-$\frac{3}{2}$ and the octet with spin-$\frac{1}{2}$. In fact *all known baryons can be assigned to either 56_{fs} or 70_{fs} representations.*

[‡]I use the subscript fs in order not to mix it up with another $SU(6)_{\mathrm{cs}}$ built of color×spin, which will be useful later.

What happened to the 20_{fs}? This representation goes with a totally antisymmetric space wavefunction, and could occur among the positive parity baryon resonances, which are quite well studied. The $SU(3)_f\times$spin content of the 20_{fs} is $|8_f 2_s\rangle$ and $|1_f 4_s\rangle$. This is the only occurrence of a spin-$\frac{3}{2}$, $SU(3)_f$ singlet. So observation of a spin $\frac{3}{2} - \Lambda^{*0}$ without the partners required to form an octet would be the signature of the 20_{fs}. There is no serious candidate for a $1_f 4_s$ state in the baryon spectrum. The QCD Breit interaction predicts this. Two quarks in the 20_{fs} must be antisymmetric in simultaneous exchange of flavor and spin. In contrast, quark pairs in the 56_{fs} must, and in the 70_{fs} may, be symmetric in flavor$\times$spin. So the 20_{fs} alone decouples from the most attractive configuration, $|\bar{3}_f 1_s \bar{3}_c\rangle$. Because of this, the 20_{fs} is promoted to higher energy and is less stable.

4 Mesons

The generalization of the Breit interaction to $\bar{q}q$ is simple. Once $\bar{q}q$ form a color singlet the residual interaction is attractive for spin-0 and repulsive for spin-1, much like positronium. The consequences are well known: The pseudoscalar mesons like the π are lighter than their vector partners like the ρ.

The fact that colorspin has other significant consequences for meson spectroscopy is less well known.

4.1 The Absence of Exotics

The absence of $SU(3)_f$ representations other than 1_f and 8_f for mesons, and 1_f, 8_f, and 10_f for baryons was one of the original inspirations for Gell-Mann's and Zweig's quark model. Later, QCD and confinement explained the absence of representations of with fractional charge, but quark model exotics like a meson 27_f-plet ($\bar{q}^2 q^2$) or a baryon $\overline{10}_f$-plet ($q^4\bar{q}$) are allowed by confinement. Gell-Mann's original paper on the quark model begs the question with a breathtakingly appropriate typographical error:

> Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming [*sic*] that the lowest baryon configuration (qqq) gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\bar{q})$ similarly gives just **1** and **8**.[10]

and almost everyone has been assuming it ever since.

The experimental situation has changed little since Gell-Mann's day. Consider, for example, states of two pions at low energies. The $I = 2$ state, exemplified by $\pi^+\pi^+$ is exotic. It contains at least $\bar{d}^2 u^2$. Data on $\pi^+\pi^+$ scattering shows no resonance at low energies, only a weak repulsion. The $I = 1$ and $I = 0$ states couple to $\bar{q}q$. The $I = 1$ phase shift resonates at the ρ. The $I = 0$ phase shift shows a strong, broad attraction from threshold up through $m_{\pi\pi} \approx 1$ GeV. The same absence of striking resonances occurs in every exotic channel.

QCD is consistent with the absence of strongly bound mesons made of more than $\bar{q}q$. Confinement saturates at q^3 or $\bar{q}q$. There are no van der Waals forces in QCD because there are no massless hadrons. So the mechanism that forms molecules from atoms in QED is absent in QCD. However, this does not explain why no $\bar{q}^2 q^2$ states have been seen.

The QCD Breit interaction, $\mathcal{H}_{\text{eff}}$, gives a simple explanation for the absence of exotics. A complete treatment would require me to include $\bar{q}q$ interactions as well as qq. However, the gist of the argument can be read off Table 1. Exotic $SU(3)_{\text{f}}$ representations require either the quarks to be in the 6_{f} or the antiquarks to be in the $\bar{6}_{\text{f}}$ representation or both. If the quarks are in the $\bar{3}_{\text{f}}$ and the antiquarks are in the 3_{f}, then the resulting $\bar{q}^2 q^2$ state transforms as an $8_{\text{f}} \oplus 1_{\text{f}}$, which is not exotic and could mistakenly be interpreted as a $\bar{q}q$ multiplet. Table 1 shows that colorspin interactions are repulsive in the 6_{f} and $\bar{6}_{\text{f}}$ representations. So colorspin pushes exotic flavor states of $\bar{q}^2 q^2$ up above thresholds where they fall apart into the light pseudoscalar mesons from which they are built. A more sophisticated treatment including $\bar{q}q$ colorspin forces confirms this result.[3]

So the QCD Breit interaction explains the absence of prominent exotic meson resonances and also suggests that we look carefully at ordinary mesons to see if any $\bar{q}^2 q^2$ states have been mistakenly cataloged as $\bar{q}q$.

4.2   The Scalar Mesons

The lightest (negative parity) light quark mesons have been known for forty years. The first excited states (with positive parity) are also well known. Of these, only the scalar mesons remain controversial. The rest were discovered and classified during the sixties and seventies. The identification and nature of the scalar mesons continues to be one of the most controversial subjects in hadron spectroscopy. The special role of the $\pi\pi$ s-wave in chiral dynamics and in nuclear physics makes this an important issue.

To make $J^{PC} = 0^{++}$ in the naive quark model it is necessary to add a unit of orbital angular momentum to a $\bar{q}q$ pair (to get positive parity).
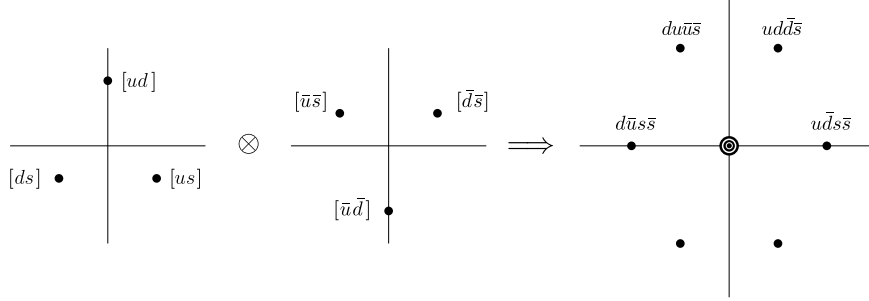
Figure 5. $SU(3)_f$ weight diagrams for two antisymmetrized quarks, two antisymmetrized antiquarks, and for the flavor nonet contructed from them.

Apparently this costs more than half a GeV in mass, since well established mesons with this makeup ($1^{++}, 1^{+-}$, and 2^{++}) are all known to lie between 1.2 and 1.6 MeV. In contrast $\bar{q}^2 q^2$ can couple to 0^{++} without excitation. Surprisingly there are striking experimental effects in all flavor nonet 0^{++} channels at masses below 1 GeV, and they do not fit naive $\bar{q}q$ quark model expectations. Recently Black et al. reviewed the situation and concluded that the light 0^{++} mesons look more like $\bar{q}^2 q^2$ than $\bar{q}q$, an idea originally suggested on the basis of the QCD colorspin interactions back in the 1970's.[3,5]

The QCD Breit interaction between quarks strongly favors the $|\bar{3}_f 1_s \bar{3}_c\rangle$ configuration for quark pairs. The $\bar{q}q$ interaction, which we have not discussed, mixes in a significant amount of the second most attractive configuration, $|\bar{3}_f 3_s 6_c\rangle$, which, notice, is also a flavor $\bar{3}_f$. Thus the lightest $\bar{q}^2 q^2$ mesons have the flavor structure $(\bar{q}^2)^{3_f}(q^2)^{\bar{3}_f}$, which leads to an octet plus a singlet ($\equiv$ a nonet). Diagonalization of the $\bar{q}^2 q^2$ generalization of $\mathcal{H}_{\text{eff}}$ in the $\bar{q}^2 q^2$ sector drives a single 0^{++} flavor nonet to remarkably low mass (600–1000 MeV in the Bag Model). Thus the *only* light meson channel that has defied classification for the past forty years is the one in which colorspin considerations leads one to expect $\bar{q}\bar{q}qq$ to be important.

Figure (5) shows how a nonet can be constructed from two quarks in a $\bar{3}_f$ and two antiquarks in a 3_f. The quark flavor content of some of the states are shown in the figure. The most striking feature of a $\bar{q}^2 q^2$ nonet is an *inverted mass spectrum*. A standard, magically-mixed $\bar{q}q$ nonet has a degenerate isosinglet and isovector at the bottom, a strange isodoublet in the middle, and a lone isosinglet at the top. This ordering is easily understood by counting the number of strange quarks in the meson. A $\bar{q}^2 q^2$ nonet is opposite: the $\bar{q}^2 q^2$ isotriplet and one of the isosinglets contain hidden strange quarks, $\{u\bar{d}s\bar{s}, \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})s\bar{s}, d\bar{u}s\bar{s}\}$ and $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})s\bar{s}$, and therefore lie at the top
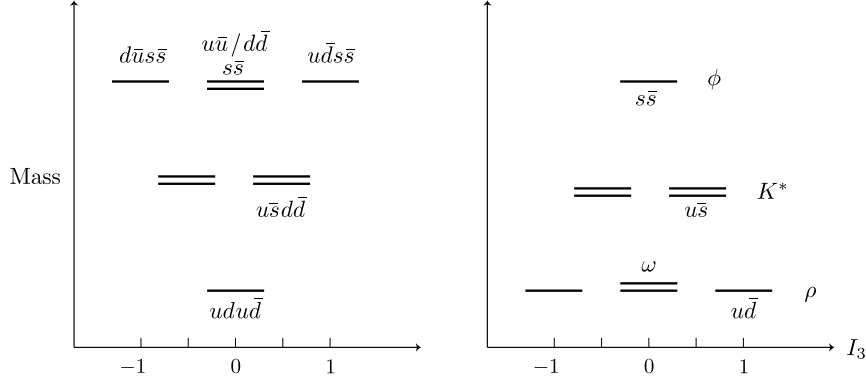
Figure 6. A cartoon representation of the masses of a $\bar{q}\bar{q}qq$ nonet compared with a $\bar{q}q$ nonet.

of the multiplet. The other isosinglet, $u\bar{d}d\bar{u}$ is the only state without strange quarks and therefore lies alone at the bottom of the multiplet. The strange isodoublets should lie in between. In summary, one expects a degenerate isosinglet and isotriplet at the top of the multiplet and strongly coupled $\bar{K}K$, an isosinglet at the bottom, strongly coupled to $\pi\pi$, and a strange isodoublet coupling to $K\pi$ in between. Fig. (6) shows the mass spectrum and quark content of a $\bar{q}^2q^2$ nonet and, for comparison, a $\bar{q}q$ nonet like the vector mesons (ρ, ω, and ϕ K^*).

The most well established 0^{++} mesons are the $I = 0$ $f_0(980)$ and the $I = 1$ $a_0(980)$. The f_0 couples to $\pi\pi$ and $\bar{K}K$. The a_0 couples to $\pi\eta$ and $\bar{K}K$. Both are so close to the $\bar{K}K$ threshold at 990 MeV that their shapes are strongly distorted by threshold effects, but it is clear that they couple strongly to $\bar{K}K$. This has always troubled those who would like to identify them as $\bar{q}q$ states. It is a natural consequence of the hidden $\bar{s}s$ component if they are predominantly $\bar{q}\bar{q}qq$.[§]

The contribution of Black et al. centers on the other isosinglet and strange isodoublet needed to fill up a nonet. If they are predominantly $\bar{q}^2q^2$ states, they should couple strongly to $\pi\pi$ and $K\pi$ respectively. Since the $\pi\pi$ and $K\pi$ thresholds are very low, one expects these states to "fall apart" into $\pi\pi$ or $K\pi$ with very large width. Enhancements in $\pi\pi$ and $K\pi$ s-wave scattering have been known for decades. Black et al. make the case that these enhancements correspond to broad states at approximately 560 MeV in $\pi\pi$ (known as the $\sigma(560)$) and at 900 MeV in $K\pi$ (known as the $\kappa(900)$). Together the

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[§]Another interpretation of the f_0 and a_0 as "$\bar{K}K$ molecules" is closely related to the $\bar{q}\bar{q}qq$ interpretation.[11]

$f_0(970)$, $a_0(980)$, $\sigma(560)$, and $\kappa(900)$ make a nonet with mass spectrum, decay couplings and widths that look qualitatively like a $\bar{q}\bar{q}qq$ system. Should this assignment hold up, it will be striking confirmation of the role of the QCD Breit interaction in hadron spectroscopy.

5 Back to Basics: Spectroscopic Rules for Q^N

In Section 2 I looked at the colorspin force between pairs of quarks. Here I will generalize that analysis to the case of N quarks all in the same spacial orbital. This allows us to look at systems of up to 18 quarks (3 colors $\times$ 3 flavors $\times$ 2 spins) at short distances. It will allow us to draw interesting qualitative conclusions about baryons and "dibaryons" (i.e., 6-quark systems).

For N quarks,

$$\mathcal{H}_{\text{eff}}^N \propto -\sum_{i\neq j}^N \{\underset{\sim}{\lambda}\vec{\sigma}\}_i \cdot \{\underset{\sim}{\lambda}\vec{\sigma}\}_j\,. \tag{9}$$

Here I have grouped the color and spin operators of the i$^{\text{th}}$ quark together. The 24 matrices $\{\underset{\sim}{\lambda}\vec{\sigma}\}$ together with the 3 Pauli matrices $\{\vec{\sigma}\}$ and the 8 Gell-Mann matrices $\{\underset{\sim}{\lambda}\}$ together form the 35 generators of an $SU(6)_{\text{cs}}$ symmetry that we can call "colorspin". Let us denote the colorspin generators as a 35-dimensional vector of 6$\times$6 matrices (in the fundamental representation), $\{\mu^r, r = 1,\ldots,35\}$. The quadratic Casimir operator of $SU(6)_{\text{cs}}$ is defined as the sum of the squares of the colorspin generators,

$$C_6^N = \sum_{r=1}^{35}\Big(\sum_{i=1}^N \mu_i^r\Big)^2 \tag{10}$$

in analogy to the total spin

$$C_2^N = 4S_N(S_N + 1) = \sum_{k=1}^3\Big(\sum_{i=1}^N \sigma_i^k\Big)^2 \tag{11}$$

and total color,

$$C_3^N = \sum_{a=1}^8\Big(\sum_{i=1}^N \lambda_i^a\Big)^2. \tag{12}$$

Of course the color Casimir, C_3^N, is zero for any physical, i.e., color singlet, hadron.

It is an elementary exercise in matrix algebra to rewrite $\mathcal{H}_{\mathrm{eff}}^N$ in terms of N, S_N, and C_6^N,

$$\mathcal{H}_{\mathrm{eff}}^N \propto 8N - \tfrac{1}{2}C_6^N + \tfrac{4}{3}S_N(S_N+1)\,. \tag{13}$$

The Casimir of $\mathrm{SU}(6)_{\mathrm{cs}}$ is the sum of 35 normed generators. S_N is the sum of only 3 identically normed generators. Therefore C_6^N dominates over S_N in eq. (13). So the spectroscopic consequences of $\mathcal{H}_{\mathrm{eff}}^N$ for Q^N states are very simple. It is important to remember that we are assuming that all quarks are in the same orbital of some mean field and that their masses can be ignored.¶

- In this approximation, the Casimirs of spin, flavor, color and colorspin all commute with $\mathcal{H}_{\mathrm{eff}}^N$, so mass eigenstates are (color singlet) states in irreducible representations of $SU(3)_{\mathrm{f}}$, $SU(6)_{\mathrm{cs}}$, and spin.

- For a given number of quarks, the largest C_6^N is associated with the most symmetric representation – the most "horizontal" Young Diagram. Since quarks in the same orbital must be antisymmetrized in color×spin×flavor, the most symmetric colorspin goes with the most antisymmetric flavor. Thus the lightest state of N quarks is the one *most antisymmetric in flavor* subject to the constraint that the associated (conjugate) colorspin representation must contain a color singlet.

- The factor of $S_N(S_N + 1)$ is generally too small to affect the order of states.

In short: Nature prefers flavor antisymmetry. It is illuminating to apply this principle to the well-known case of $N = 3$ and then to the more speculative case of $N = 6$.

6 Baryons and Dibaryons

The simple rule for the spectrum of N quarks has interesting consequences for $N = 6$. First, however, let us see how it works for the baryons, $N = 3$.

6.1 Baryons Reconsidered

The allowed colorspin and flavor-irreducible representations are tabulated in terms of Young Diagrams in Fig. (7). The flavor $SU(3)_{\mathrm{f}}$ representations, 1_{f},

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¶Ignoring the s-quark mass is not a good approximation. Fortunately s-quark mass effects can be calculated perturbatively.

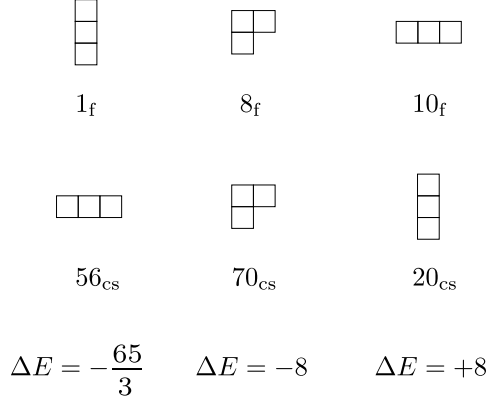$$1_{\mathrm{f}} \qquad\qquad 8_{\mathrm{f}} \qquad\qquad 10_{\mathrm{f}}$$

$$56_{\mathrm{cs}} \qquad\qquad 70_{\mathrm{cs}} \qquad\qquad 20_{\mathrm{cs}}$$

$$\Delta E = -\frac{65}{3} \qquad \Delta E = -8 \qquad \Delta E = +8$$

Figure 7. The states of three quarks, classified in $SU(6)_{\mathrm{cs}} \otimes SU(3)_{\mathrm{f}}$. For each multiplet the colorspin and flavor Young Diagrams are shown. To make states antisymmetric under flavor×color×spin, quarks must have the conjugate permutation symmetry under flavor and colorspin, obtained by interchanging rows and columns in the Young Diagram. The Breit Interaction favors the most antisymmetric flavor configuration, as evidenced by the eigenvalues displayed with the diagrams.

8_{f}, and 10_{f} are paired with the colorspin $SU(6)_{\mathrm{cs}}$ representations 56_{cs}, 70_{cs}, and 20_{cs}, respectively. The "flavor antisymmetry" rule dictates

$$M_1 < M_8 < M_{10} . \tag{14}$$

In fact the expectation values of $\mathcal{H}_{\mathrm{eff}}^{N=3}$ in the three states are $\langle\mathcal{H}_{\mathrm{eff}}\rangle_1 = -65/3$, $\langle\mathcal{H}_{\mathrm{eff}}\rangle_8 = -8$, and $\langle\mathcal{H}_{\mathrm{eff}}\rangle_{10} = +8$. To complete the classification we must find out which of these colorspin multiplets contain color singlets. The $SU(3)_{\mathrm{c}} \times SU(2)_{\mathrm{s}}$ content of $SU(6)_{\mathrm{cs}}$ representations is best known in the context of Gursey's old $SU(6)_{\mathrm{fs}}$. We can copy those results over for color×spin:

$$56_{\mathrm{cs}} \supset 8_{\mathrm{c}}2_{\mathrm{s}} \oplus 10_{\mathrm{c}}4_{\mathrm{s}}$$
$$70_{\mathrm{cs}} \supset 1_{\mathrm{c}}2_{\mathrm{s}} \oplus 8_{\mathrm{c}}2_{\mathrm{s}} \oplus 8_{\mathrm{c}}4_{\mathrm{s}} \oplus 10_{\mathrm{c}}2_{\mathrm{s}}$$
$$20_{\mathrm{cs}} \supset 1_{\mathrm{c}}4_{\mathrm{s}} \oplus 8_{\mathrm{c}}2_{\mathrm{s}} \tag{15}$$

A novel perspective on the QCD picture of baryons emerges. The most attractive colorspin channel, which would be a candidate for the lightest baryon, the flavor singlet 56_{cs}, is not a physical state because it does not contain a color singlet. So the colorspin rule to antisymmetrize flavor is, in this case, frustrated by Fermi statistics: a color singlet, flavor singlet three quark state would have to be antisymmetric in spin, which is not possible for spin-$\frac{1}{2}$. Otherwise, the baryon ground state would be a flavor singlet, Λ-like *uds* baryon.
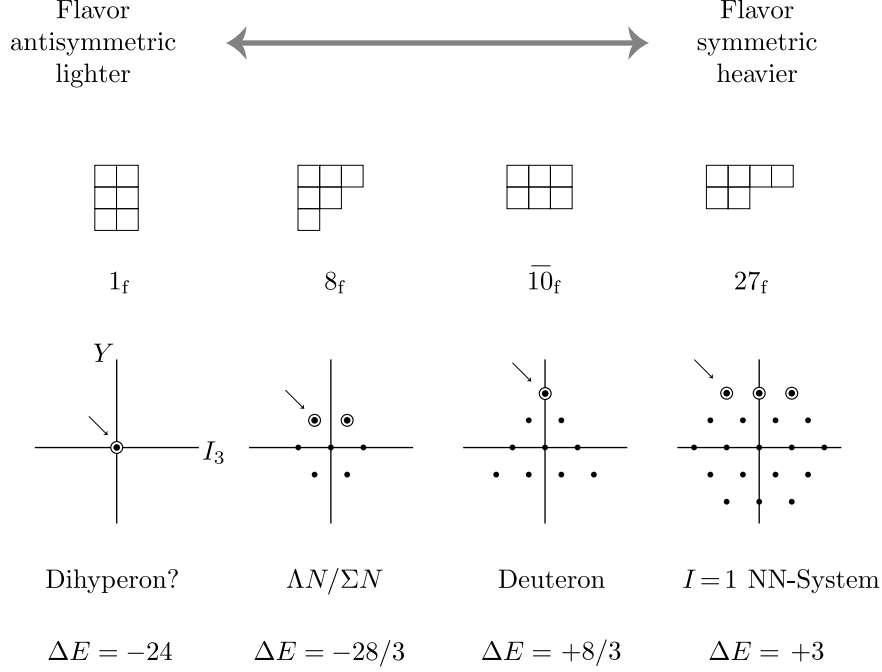
| Flavor antisymmetric lighter | | | Flavor symmetric heavier |

| 1_f | 8_f | $\overline{10}_\mathrm{f}$ | 27_f |

| Dihyperon? | $\Lambda N/\Sigma N$ | Deuteron | $I\!=\!1$ NN-System |

| $\Delta E = -24$ | $\Delta E = -28/3$ | $\Delta E = +8/3$ | $\Delta E = +3$ |

Figure 8. Some of the states of six quarks, classified in $SU(6)_\mathrm{cs} \otimes SU(3)_\mathrm{f}$. Only the flavor Young Diagrams are shown. The colorspin diagrams are conjugate. Again, the most antisymmetric possible flavor state has the most attractive colorspin interaction. The multiplets shown are interesting because the correspond to physically interesting channels: the H, the $\Lambda N/\Sigma N$ system, the deuteron, and the isotriplet dinucleon, respectively.

The flavor-antisymmetry rule works fine for the rest of the three quark states; the mixed symmetry multiplet, the 70_cs, is lightest. It contains a color singlet with spin-$\frac{1}{2}$ – the familiar flavor octet baryons. The antisymmetric colorspin multiplet, the 20_cs, is heaviest. It contains a color singlet with spin-$\frac{3}{2}$, the familiar decuplet baryons.

One final comment: It is tempting to suggest that the excluded multiplet, the 56_cs, could neutralize its color with a constituent gluon. The result would be a $qqqg$ $SU(3)_\mathrm{f}$ singlet "hybrid" baryon. There is a rather light negative-parity Λ at 1405 MeV that has often caused problems for quark modelers. However current wisdom favors a q^3 or $q^4\bar{q}$ interpretation.[12]

Now consider $N = 6$. The flavor Young Diagrams for six quarks are shown in Fig. (8). The colorspin Young Diagrams are conjugate (rows and columns interchanged). Only a few of the multiplets are shown. For the multiplets shown, the colorspin interaction energies are $\langle \mathcal{H}_{\text{eff}} \rangle_1 = -24$, $\langle \mathcal{H}_{\text{eff}} \rangle_8 = -28/3$, $\langle \mathcal{H}_{\text{eff}} \rangle_{\overline{10}} = +8/3$, and $\langle \mathcal{H}_{\text{eff}} \rangle_{27} = +3$.

The most attractive channel is the flavor singlet "dihyperon" (known as the H) with quark content $u^2 d^2 s^2$.[4] If colorspin were the only consideration, its interaction energy, -24, would bind it with respect to decay into two octet baryons (colorspin energy 2×-8). It would decay into the would-be flavor singlet baryon (colorspin energy $2 \times -65/3$), if that state were not excluded by color confinement. Models of confined light quarks are not sufficiently accurate to decide unequivocally whether the H is bound. Lattice calculations are improving and may answer the question before experimenters are able either to discover it or establish that it is not bound.[13]

The next most attractive channel is the flavor octet. The hypercharge $+1$ members have the quantum numbers of ΣN or ΛN. A strong resonance with these quantum numbers has long been known in ΛN scattering at 2129 MeV. Unfortunately this cannot be unambiguously interpreted as evidence for colorspin attraction because a loosely bound state with the same quantum numbers is expected as an $SU(3)_{\text{f}}$ analog of the deuteron.

The $\overline{10}_{\text{f}}$ and 27_{f} channels are interesting because the colorspin interaction energy is repulsive. These flavor representations are the smallest ones that contain states with the quantum numbers of the deuteron (the $\overline{10}_{\text{f}}$) and the $I = 1$ two nucleon system (the 27_{f}). Both appear at $Y = 2$ (which is strangeness zero) in the $SU(3)_{\text{f}}$ weight diagrams shown in Fig. (8). I interpret the positive colorspin interaction energy as evidence that the quark Breit interaction is repulsive at short distances in these channels. Once the three quark baryons have separated, apparently other effects generate an intermediate range attraction, which binds the deuteron and almost binds the $I = 0$ NN system.

7 Colorspin in Quark Matter

The most interesting recent application of colorspin systematics in QCD is the study of condensates and symmetries in cold, dense matter.[6] The attraction between quark-antiquark pairs is so strong in QCD that $\bar{q}q$ pairs condense in the vacuum, breaking chiral symmetry. Calculations show that the color singlet, spin-zero $(1_{\text{c}}1_{\text{s}})$ configuration is the most attractive channel for $\mathcal{H}_{\text{eff}}$.

So condensation in this channel is consistent with colorspin considerations. The $\bar{q}q$ condensate breaks the flavor $SU(3)_{fL} \times SU(3)_{fR}$ chiral symmetry spontaneously down to vector $SU(3)_f$, the standard flavor symmetry of the hadron spectrum. Interestingly, the colorspin interaction energy in the color antitriplet, spin-zero $(\bar{3}_c 1_s)$ qq channel is half as attractive. But no condensate develops in this channel in vacuo. It is forbidden by the Vafa-Witten Theorem, which says that vector symmetries cannot break spontaneously in vector gauge theories like QCD.[14]

Things change in the presence of matter.[15,6] At zero temperature and finite density, degeneracy effects become important. At some point close to where the chemical potential for baryon number (μ_B) becomes comparable to Λ_{QCD}, nuclear matter passes over to quark matter. Studies show that the attraction that drives $\bar{q}q$ condensation weakens as μ_B increases. They also show that the qq colorspin attraction in the $\bar{3}_c 1_s$ channel can generate a condensate in the vicinity of the Fermi surface similar to BCS pairing in superconductivity. Alford, Rajagopal and Wilczek argue that this "color superconductivity" is robust – it occurs both in colorspin models of the quark force and in instanton models as well. Of course, QCD at moderate densities is still strongly interacting, and these predictions must be regarded as qualitative. Authoritative statements await a lattice treatment of quark matter at finite chemical potential.

If qq condenses in the $\bar{3}_c 1_s$ channel in quark matter, color symmetry is broken from $SU(3)_c$ to $SU(2)_c$. Of the eight gluons, the three generators of color-$SU(2)_c$ remain massless. The four gluons that are doublets under $SU(2)_c$ get mass a la the Meissner effect. One linear combination of the eighth (color hypercharge) gluon and the photon stays massless (much as the standard model photon survives weak $SU(2)$ symmetry breakdown). $SU(2)_c$ nonsinglet quark or gluon excitations still experience long-range confining forces, but $SU(2)_c$ singlet states do not. Thus, for example, two colors of each quark flavor (the ones that form the $SU(2)_c$ doublet) remain confined, but the third color, say "blue", is a freely propagating fundamental excitation in the medium.

For flavor symmetries, qq condensation has interesting consequences. The two-flavor and three-flavor cases differ dramatically. In two-flavor QCD (obtained by imagining the strange quark mass to be much larger than μ_B) the $\bar{3}_c 1_s$ condensate is a $SU(2)_{fL} \times SU(2)_{fR}$ singlet so it has no effect on the flavor symmetries (though it does violate baryon number). In particular, chiral symmetry is unbroken. Since chiral symmetry is broken spontaneously in the QCD vacuum, there must be a phase transition between $\mu_B = 0$ and the high density phase where superconductivity sets in.

In three-flavor QCD (imagine the strange quark mass to be negligible compared to μ_B) the qq condensate is in the antisymmetric $\bar{3}_c 1_s \bar{3}_f$ channel. Thus the condensate breaks both color and flavor $SU(3)$ symmetries. However, the condensate is invariant under *simultaneous* rotations in color and flavor, which means that a "diagonal" vector $SU(3)$ symmetry remains unbroken. This is the same flavor structure as the low density phase (only a vector $SU(3)_f$ remains unbroken in vacuo with three massless quarks), so it is possible that there is no phase transition from the low density to high density phase. Dubbed "color-flavor locking",[16] this phenomenon leads to several interesting speculations on the phase structure of QCD at finite densities. Implications for observations at RHIC and for the behavior of neutron stars are the subject of much recent interest.[17,18]

With this I've come to the end of this tour of the phenomenological implications of the Breit interaction in QCD. Insights into QCD in the confining domain are precious. The underlying Lagrangian is so simple, the phenomena are so rich, and the theoretical structure is so complex. It is remarkable that so much can be understood qualitatively in terms of the simple effective interaction,

$$\mathcal{H}_{\text{eff}} \propto -\sum_{i \neq j} \underset{\sim}{\lambda}_i \cdot \underset{\sim}{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \tag{16}$$

whose origins go back to Gregory Breit in the first half of the twentieth century.

Acknowledgments

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